

WORKSHOP ON DATA SCIENCE AND EDUCATION INTERNATIONAL CONFERENCE ON DATA SCIENCE ICDS 2023 CHILE

Classification in educational data: Cognitive Diagnostic Models using different R packages

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Santiago, November 7, 2023

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- **1. Motivation: A practical example**
- 2. A short literature review
- 3. Modeling
- 4. Estimation Methods and R packages
- 5. Results for BDI data
- 6. Comments
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MOTIVATION: A PRACTICAL EXAMPLE

 1111 students from the University of Sâo Paulo completed the test of depression of Beck (Beck et al, 1981).

- This test have 21 questions of four alternatives (0 to 3 points).
- In the traditional approach (See Kendall et al, 1987), a score is obtained adding the responses of the questions. The minimal score is 0 in the maximum is 63.
- Scores are used to do comparisons and can be obtained using Classical Test Theory or Item Response Theory.
- However, in order to give a best meaning is necessary classify the individuals in groups in relation with the depression

In the traditional approach individuals are classified as being :

- non depressed (BDI total score 0-15),
- dysphoric (BDI total score 16-20),
- depressed subjects (BDI total score 21-63).
- But which is the meaning of score 21 and why it is depressed but score 20 is dysphoric?
- It looks qualitative but the cut-off score could be arbitrary.
- The same problem appear in educational assessment. We have scores of Math or Reading and it is useful to comparison but as educators we really need to classify students to define policies and learning activities.

• We need a new paradigm in Psychometric.

- In recent years, cognitive diagnostic models (CDMs) has emerged as a new psychometric paradigm capable of providing meaningful diagnostic feedback.
- CDMs allows the classification of examinees in multiple cognitive attributes. This measurement is obtained by modelling these attributes as categorical, discrete latent variables.
- The latent variable may be a cognitive skill (say, mathematics achievement), a psychological trait, or an attitude.
- In this talk we adopt a practical view, showing as use different estimation methods using R packages



A SHORT LITERATURE REVIEW

Collares (2022):

- The origins of cognitive diagnostic modelling can be traced back to 1983 with the rule space method (Tatsuoka, <u>1983</u>).
- In a further development of the rule space method, Birenbaum et al. (<u>1992</u>) developed the Q-matrix: a way to arrange items according the necessary "rules" (i.e., cognitive attributes) needed to solve them

In general, CDMs or diagnostic classification models allows the classification of examinees in multiple skills or cognitive attributes.

These models are relatively newer psychometric framework for collecting, analyzing, and reporting diagnostic data. They are a third generation of models in Psychometric after Theory Classical of the Test (TCT) and Item Response Theory (IRT).

CDMs have received increasing attention in many disciplines, such as educational, psychological, and psychiatric measurement and different models are being proposal attending the different formats of response of the Test how dichotomous, polytomous, count and continuous response. This models are important because there is a real interest in developed formative assessments to provide examinees (students) and evaluators (teachers) with detailed feedback on what examinees (students) what skills they have (are able to do) yielding information that can optimize counseling (instruction) and improvement (learning).

In other words, a formative assessment should identify individual strengths and weaknesses in a particular content, which results in enhanced teaching and learning environment (DiBello & Stout, 2007).

 CDMs can provide test takers with specific feedback on their strengths and weaknesses, and hence CDM applications go beyond a simple ranking or locating individuals in relation to an underlying latent trait.

• This model is commonly estimated under a frequentist approach using Maximum Likelihood (ML) estimation methods since that Bayesian estimation considering MCMC methods are usually slow for large data sets.



Theory, Methods, and Applications

André A. Rupp Jonathan Templin Robert A. Henson Methodology of Educational Measurement and Assessment

Matthias von Davier Young-Sun Lee *Editors*

Handbook of Diagnostic Classification Models

Models and Model Extensions, Applications, Software Packages

Description Springer



MODELING

Fragoso and Curi (2013) dichotomize the data, such that the value 0 was attributed to the answers equal to zero, and a value of 1 was attributed to positive answers (1, 2, or 3).

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													BDIda	ata												
1	n	Gender	Age	BDI1	BDI2	BDI3	BDI4	BDI5	BDI6	BDI7	BDI8	BDI9	BDI10	BDI11	BDI12	BDI13	BDI14	BDI15	BDI16	BDI17	BDI18	BDI19	BDI20	BDI21		
2	1	2	22	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0		
3	2	2	33	1	1	1	1	1	0	1	1	0	0	1	1	1	0	1	0	0	0	0	0	0		
4	3	2	25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
5	4	2	20	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
5	5	1	21	1	0	0	1	1	1	1	1	0	0	1	0	0	1	1	0	1	0	0	0	0		
7	6	1	25	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	1	1	0	0	0	1		
В	7	1	22	1	1	0	0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	0	1	0		
9	8	2	21	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0		
0	9	1	20	0	1	0	1	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0		
1	10	2	21	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0		
2	11	1	21	1	1	1	1	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0		
3	12	1	21	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0		
4	13	1	23	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0		
5	14	2	24	0	0	0	1	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0		
6	15	2	23	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0		
7	16	1	21	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0		
8	17	1	20	0	0	1	0	1	0	0	0	0	0	1	1	0	1	1	0	1	1	0	0	0		
9	18	1	19	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
20	19	1	23	1	0	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	0	0	1	0		

- For BDI data we can use an Item Response Theory (IRT) models used for identify latent trait and item parameters. Concerning to the respondents, in IRT models, the primarily intent is ranking individuals; We want rank the Depression's individuals?
- Other possibility is use the Cognitive Diagnostic Models (CDM) where the intent is classifying individuals as possessing or not a skill or characteristic of the Depression;





 In IRT models the performance of the individual is based in a continuous latent trait. Then, individual with higher latent trait have higher probability to answer correctly the item.

- In CDM models the performance of the individual is based in discrete latent trait (attributes). Then, individual which has all skills defined in one item have higher probability to answer correctly the item.
- The map the attributes necessary for responding correctly to each question on a test; this map is called the **Q**-matrix.

- In IRT, the probability of correct response is affected for two kind of latent factors. The first is associated with the individual (Trait latent) and the other is associated wit the item (item parameters).
- In CDM, the probability of correct response is affected for the latent response of the individual for the item and the item parameters. The latent response is affected for two kind of factors. The first is a latent factor associated with the skill of the individual and the other is the specification of skills in the item.

3.2. CDMs

- There are several different approaches to the modeling using CDM. A good initial revision can be seen in George and Robitzsch (2015), but since then more models are being developed each year;
- The non compensatory deterministic input noisy-and gate (DINA; Haertel 1989; Junker and Sijtsma 2001) model.
- The compensatory deterministic input noisy-or-gate (DINO; Junker and Sijtsma 2001) model,
- The generalized version (G-DINA; de la Torre 2011)
- Others "ACDM", "LLM", "RRUM", and "MSDINA".
- Versions of the models to Dichotomous, Polytomous and Continuous responses

3.3. DINA model

- One of the most popular models in the CDM class is the Deterministic Input Noisy ``and" gate, due to its good performance and easiness of interpretation.
- To understand the model, it is important to define some quantities for the input. We have:
 - i = 1, ..., N respondents to a questionnaire;
 - j = 1, ..., J items to be responded;
 - k = 1, ..., K skills (or dimensions) to be evaluated.



- Each individual have a *skill profile*, which is the vector containing the possessing of skills of that individual $\alpha_i = (\alpha_{i1}, ..., \alpha_{iK})$ which is considered latent. In this case are K skills. It is a latent variable that we want to know.
- Each item j=1,...,J of the test can evaluate one or more attributes (skills) on the test.
- Each skill k=1,..., K, can be measure for different items. The matrix what contains, in each row, information about which skills are evaluated by which item is named Q matrix is assumed known.

 Fragoso and Curi (2013) given the following distribution of the items in the two dimensions identified in the BDI Test





- We have J=21 items
- The items of the BDI test measure two skills, cognitive and somatic-affective. Some items measure one and other measure both. The figure before is the Q-matrix.
- We are interested in know the profile of the individual i

 $\alpha_i = (\alpha_{i1}, \alpha_{i2}).$

- There is three possibilities, {(0,0), (1,0), (0,1), (1,1)}.
- In the first case there is no depression, on the second we say that the depression of the individual is cognitive, on the third case it is somatic-affective and the last case the individual present both characteristics (skills) of the Depression.

Other example

- Take by example a Grade Level Assessment Test. End of 6th grade. This test can evaluate different aspects concerning to the knowledge of Math;
- The test to evaluates three different skills that the students will had:
 1) Reading, 2) English and 3) Math; It is *K* = 3
- Each item j = 1, ..., J of the test can evaluate only one of the attribute (skills) or more than one simultaneously.

In the Test example, if an item j evaluates the possessing of the two first skills but no the last (Reading, English but no Math), the row of the that item in the **Q**-matrix will be $\mathbf{q}_j = (1,1,0)$;

- The Q-matrix can be defined by a group of experts in the field of the assessment or using automated procedures. However, recently there is contributions for made proposing different algorithms.
- In the example, we are interested in know if the student can be classified in any of the 8 groups possible: (0,0,0), (1,0,0), (1,1,0), (0,1,1), (1,0,1), (1,1,1).

Specification of Q-matrix is very important!! Here some works

- Chen, Y., Liu, J., Xu, G., and Ying, Z. (2015). Statistical analysis of qmatrix based diagnostic classification models. *J. Am. Statist. Assoc.* 110, 850–866
- de la Torre, J., and Chiu, C.-Y. (2016). A general method of empirical Q-matrix validation. *Psychometrika* 81, 253–273.
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- Köhn, HF. & Chiu, CY. (2018) How to Build a Complete Q-Matrix for a Cognitively Diagnostic Test. *Journal of Classification* 35(2): 273-299.
- Gao, M., Miller, M. D., & Liu, R. (in press). The impact of Q-matrix misspecification and model misuse on classification accuracy in the generalized DINA model. *Journal of Measurement and Evaluation in Education and Psychology.*



By considering α_i and \mathbf{q}_j above, we can define a latent response variable η_{ij} for the

jth item in the ith individual as

$$\eta_{ij} = \prod_{k=1}^{K} \alpha_{ik}^{q_{jk}} = 11(\boldsymbol{\alpha}_{i}' \mathbf{q}_{j} = \mathbf{q}_{j}' \mathbf{q}_{j}),$$

where $11(\cdot)$ denoting the indicator function. Here, η_{ij} indicates if the *i*th individual has the skills demanded by the *j* th item or not.



• In the Test example, consider the individual with the following latent profile $\alpha_i = (0,0,1)$ (only has Math skills). which answer the item j with the following information $\mathbf{q}_j = (1,1,0)$ indicating that this item measure the skills of Reading and English. Then

 $\eta_{ij} = \alpha_{i1}^{q_{j1}} \times \alpha_{i2}^{q_{j2}} \times \alpha_{i3}^{q_{j3}} = (0)^1 \times (0)^1 \times (1)^0 = 0$ indicate what the individual *i* has not the skills required in the item *j*.

The student have not the skills of Reading and English measured on the test.

Another important thing is to define the format of the answers;

- In usual DINA Model the answers need to be dichotomous, that is, correct or incorrect, yes or no, agree or disagree, etc.
- There is also a DINA Model for polytomous answers (Tu et. al., 2017), which is useful for agreement tests, allowing the researcher to evaluate the degree of agreement;
- Recently a DINA Model for continuous responses was proposed (Minchen et. al, 2017), allowing the researcher to use questionaries with this kind of answers or latent traits such as the time to respond to an item;

• Our study is based in the dichotomous case





For dichotomous answers we will have the following item parameters for the item j:

- The probability of ``guessing'', that is, getting a right answer to an item the individual does not possess the skills to answer correctly

$$g_j = P(Y_{ij} = 1 | \eta_{ij} = 0)$$

- The probability of ``slipping", that is, answering wrongly an item the individual possess the skills demanded by it;

$$s_j = P(Y_{ij} = 0 | \eta_{ij} = 1)$$



W. Gan, Y. Sun, S. Ye, Y. Fan and Y. Sun, "AI-Tutor: Generating Tailored **Remedial Questions and** Answers Based on Cognitive Diagnostic Assessment," 2019 6th International Conference on Behavioral, Economic and Socio-Cultural Computing (BESC), Beijing, China, 2019, pp. 1-6, doi: 10.1109/BESC48373.2019. 8963236.



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ESTIMATION METHODS AND R PACKAGES



- For DINA models is possible use Frequentist and Bayesian approach.
- R packages are available for both estimation methods (CDM, GDINA, dina)
- Additionally, is possible use R with interface for other Bayesian software as WinBUGS, JAGS or STAN (R2wingbugs,R2jags,Rstan)

Approah	R package	Method	Reference	Models	Home page
Classical or Frequentist	CDM	EM Algorithm	Robitzsch, Kiefer, George, & Uenlue, (2016)	Several	<u>https://cran.r-</u> project.org/web/packages/CDM/index.html
	GDINA	MMLE/EM algorithm	Ma and de la Torre (2019)	Several	<u>https://cran.r-</u> project.org/web/packages/GDINA/index.html
Bayesian	Dina	Gibbs Sampling	Culpepper (2015) ,Culpepper and Balamuta (2019)	DINA	<u>https://cran.r-</u> project.org/web/packages/dina/index.html
	R2BUGS; R2JAGS (WINBUS, JAGS)	Metropolis Hastin g	Zhan et al (2019)	Several	
	RSTAN (STAN)	NUTS	Silva et al (2018) submitted 2016, Lee (2017)	DINA	https://mc-stan.org/documentation/case- studies/dina_independent.html



Journal of Statistical Software

October 2016, Volume 74, Issue 2.

doi:10.18637/jss.v074.i02

Practical Assessment, Research & Evaluation

A peer-reviewed electronic journal

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Volume 20, Number 11, April 2015

ISSN 1531-7714

Cognitive Diagnostic Modeling Using R

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The R Package CDM for Cognitive Diagnosis Models

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2015 Vol. 11 no. 3

DOI: 10.20982/kpmp.11.3.p189

Cognitive Diagnosis Models in R: A Didactic

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Journal of Statistical Software

May 2020, Volume 93, Issue 14.

doi: 10.18637/jss.v093.i14



Implementation of Cognitive Diagnosis Modeling using the GDINA R Package

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GDINA: An R Package for Cognitive Diagnosis Modeling

Wenchao Ma The University of Alabama Jimmy de la Torre The University of Hong Kong

MEASUREMENT: INTERDISCIPLINARY RESEARCH AND PERSPECTIVES 2018, VOL. 16, NO. 1, 71–77 https://doi.org/10.1080/15366367.2018.1437243

SOFTWARE REVIEW

GDINA and CDM Packages in R

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JOURNAL OF EDUCATIONAL AND BEHAVIORAL STATISTICS OnlineFirst, published on August 10, 201 doi:10.3102/1076998615595403

Journal of Educational and Behavioral Statistics Vol. XX, No. X, pp. 1–23 DOI: 10.3102/1076998615595403 © 2015 AERA. http://jebs.aera.net

Bayesian Estimation of the DINA Model With Gibbs Sampling

Tutorial

Steven Andrew Culpepper University of Illinois at Urbana-Champaign Received: 31 October 2016 Revised: 18 August 2017 Accepted: 21 August 2017

EDE: 10.1002/smj.2016/D225

RESEARCH PAPER

Biometrical Journal

Estimating the DINA model parameters using the No-U-Turn Sampler

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> Journal of Educational and Behavioral Statistics Vol. XX, No. X, pp. 1–31 DOI: 10.3102/107699861982004 Article reuse guidelines: sagepub.com/journals-permissions © 2019 AERA. http://jebs.aera.net

Using JAGS for Bayesian Cognitive Diagnosis Modeling: A Tutorial

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- If you want to apply the methodology of CDM the best recommendation is use the frequentist approach, CDM and GDINA are recommendable packages and many models could be fitted using them quickly.
- If you have more interest in methodological research and then propose new models or explore variants of the previous models a good recommendation is use Bayesian approach, specially using JAGS or STAN where both could be implemented in R and Python.

Bayesian methods are more delayed than frequentist approach

- There is some important advantages when used a Bayesian approach and when an intermediary program is used as JAGS (BUGS) or STAN:
- a) Distribution of the parameters of the model and not only a punctual estimation and standard deviation assuming Asymptotic normality, it is specially relevant since that parameters in the model are in the (0,1) interval
- b) Possibility of implement easily new models,
- c) Restrictions in the model are substituted by priors and priors can include historic information and then the model is identified.



RESULTS FOR BDI DATA

 In order to adjust the DINA model for BDI data, we used a dichotomization of the answers, as proposed first by Fragoso and Curi (2013);

• The Q-matrix was constructed based on K = 2 skills, which we call dimensions in this work, for interpretation facility;

• These dimensions are based in IRT and are the cognitive (α_1) and somatic-



Estimation of item parameters

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ltom	Q (dim	nensions)	ĝ	2	ŝ	ŝ			
item	α_1	α_2	Mean	sd	Mean	sd			
1. Sadness	1	1	0.468	0.020	0.101	0.017			
2. Pessimism	1	1	0.189	0.017	0.334	0.026			
3. Sense of failure	1	0	0.045	0.012	0.444	0.024			
4. Lack of satisfaction	1	1	0.309	0.020	0.163	0.020			
5. Guilty feelings	1	0	0.039	0.011	0.423	0.024			
6. Sense of punishment	1	0	0.115	0.017	0.453	0.023			
7. Self-dislike	1	0	0.242	0.022	0.158	0.019			
8. Self-accusation	1	0	0.422	0.023	0.163	0.017			
9. Suicidal wishes	1	1	0.032	0.008	0.694	0.023			
10. Crying spells	1	1	0.142	0.014	0.502	0.026			
11. Irritability	0	1	0.283	0.024	0.279	0.023			
12. Social withdrawal	1	1	0.210	0.017	0.394	0.025			
13. Indecisiveness	1	1	0.205	0.016	0.320	0.025			
14. Distortion of body image	1	1	0.222	0.017	0.458	0.025			
15. Work inhibition	1	1	0.259	0.018	0.206	0.023			
16. Sleep disturbance	0	1	0.262	0.028	0.288	0.022			
17. Fatigability	0	1	0.348	0.030	0.162	0.019			
18. Loss of appetite	1	1	0.178	0.016	0.560	0.026			
19. Weight loss	0	1	0.062	0.012	0.851	0.016			
20. Somatic preoccupation	1	1	0.223	0.016	0.518	0.026			
21. Loss of libido	0	1	0.109	0.017	0.645	0.022			

sd: standard deviation.

Profile estimate and comparison with usual classification

6		Dime	ensions		$\widehat{\pi}$			
C		$lpha_1$	$lpha_2$		Mean	sd		
1	(non-depressive)	0	0	(0.363	0.024		
2	(symptomatic of cognitive dimension)	1	0	(0.124	0.016		
3	(symptomatic of somatic-affective dimension)	0	1	(0.124	0.021		
4	(both symptoms)	1	1	(0.389	0.019		

sd: standard deviation.

Diagnosis proposed by DINA	Groups according to usual classification							
Diagnosis proposed by DINA	Depressed	Dysphoric	Non Depressed					
Non-depressive	0(0%)	0(0%)	442(51.64%)					
Symptomatic to cognitive	0(0%)	5(4.39%)	116(13.55%)					
Symptomatic to somatic-affective	0(0%)	0(0%)	106(12.38%)					
Both symptoms	141(100%)	109(95.61%)	192(22.43%)					

- The DINA model approach in this application, consider two skills which characterize the Depression: cognitive and somaticaffective dimensions
- This dimensions were obtained using previous literature (Fragoso and Curi, 2013) considering IRT approach which was used to define a Q matrix.
- The results obtained using DINA model permit classify the examinees in four groups defining the probability of each examinee is in each group.
- The results obtained can be interpreted similarly to traditional classification using BDI scores but had some interesting different results which is useful in classifying individuals as part of diagnostic of depression.

 However, it is notable that using this approach may overestimate depression, mainly because the dichotomization used causes all positive responses to an item to have the same weight in final diagnostics.

- Our example with BDI items is not a direct proposal to clinical use, It has the intention of showing the kind of data DINA model fits and to motivate further studies with the possibilities brought by this methodology.
- Similar examples can be use in Education identifying the skills that the students can do offering a best interpretation of the results of Assessment.



COMMENTS

- With the already existent models and the one to be proposed, it is possible to evaluate many kinds of questionnaires;
- The outputs are interesting both for evaluating the items and the respondents;
- To run applications using CDM to an assessment it is important to define skills (or dimensions) evaluated by each item of a test and use Q matrix well defined;
- Possible applications can be done in many study fields such as education, psychology, sociology and others.

Install packages dina, CDM, GDINA and R2jags from the repository in R and dependences. Install JAGS from https://sourceforge.net/projects/mcmc-jags/files/

Files

- BDIdata.csv contain the date a set of BDI test, 21 items and 1111 individuals, dichotomous responses.
 - ScriptDina.R script to analyze the data in R
 - ScriptDina.Rmd script to create a report using Rmarkdown in RStudio. This file depends on the following files and of BDIdata.csv:
 - ScriptDina.RData file with the image of the run of ScripDina.R. Contain the results what will called by the file ScriptDina.Rmd, abnt.csl and refs.bib has respectively, some functions of Brazilian Portuguese and references cited on the report
 - ScriptDina.html is the report.

<u>Scripts</u>

<u>Report</u>

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Thank you for your attention!

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